UNIVERSITÀ DEGLI STUDI "G. D'ANNUNZIO" DI CHIETI-PESCARA
FACOLTÀ DI ARCHITETTURA

CORSO DI LAUREA SPECIALISTICA, CORSI DI LAUREA TRIENNALI

c.i. **SCIENZA DELLE COSTRUZIONI e TEORIA DELLE STRUTTURE**

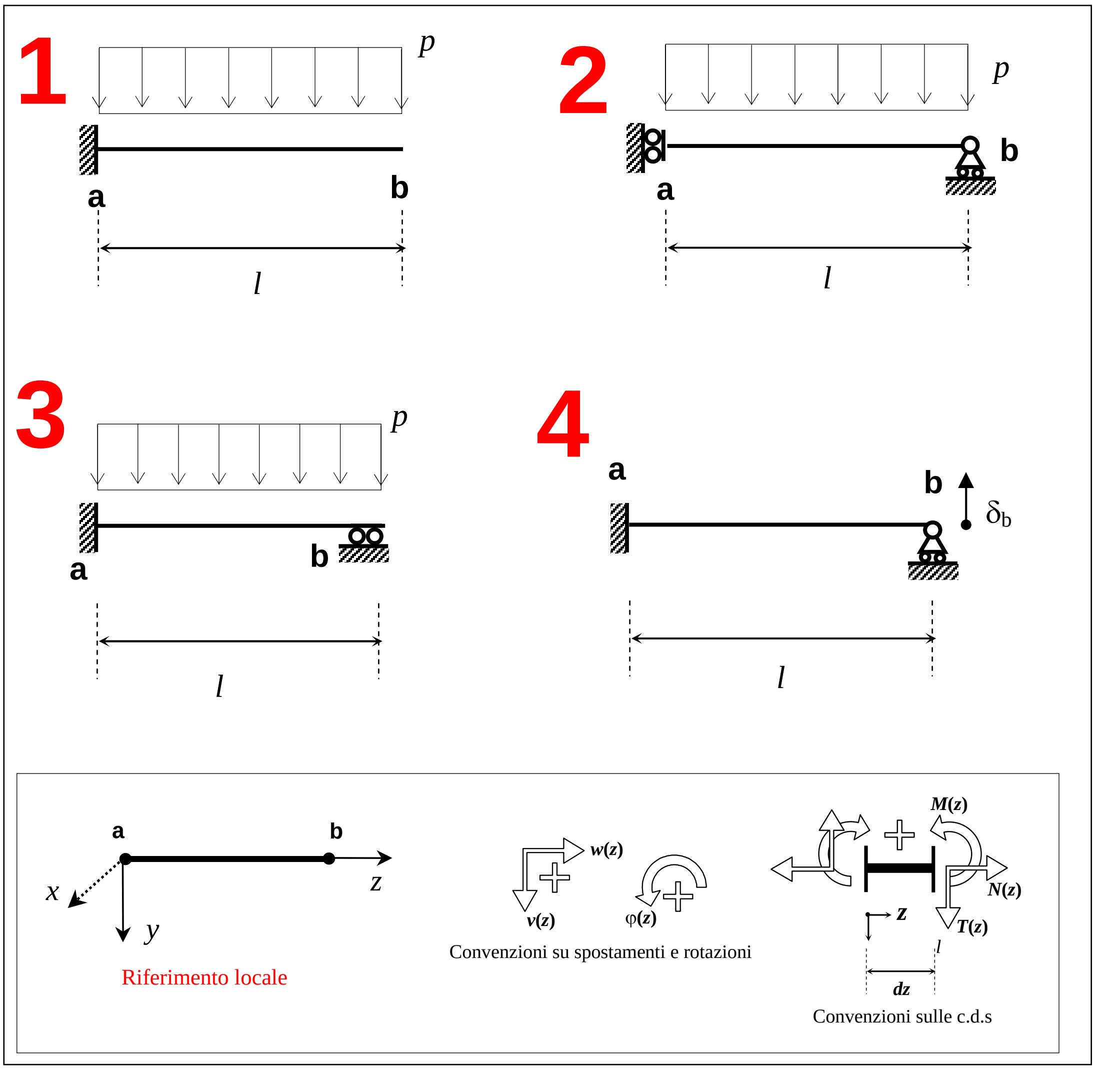
a.a. 2009-2010

Marcello Vasta, Paolo Casini

Esercizi di preparazione alla prova d'esonero

Linea Elastica

Problemi 1-4 Per ciascuna delle *travi indeformabili al taglio* riportate in figura si chiede di: **a)** scrivere l'equazione della linea elastica con le rispettive condizioni al contorno; **b)** determinare la legge di variazione della funzione $v(z)$ e diagrammarne qualitativamente l'andamento (*linea elastica* o *deformata* della trave); **c)** determinare le leggi di variazione del taglio e del momento flettente, ricordando che: $T(z) = -EI\, v'''(z)$, $M(z) = -EI v''(z)$.



COGNOME...

NOME..

MAT. ....

SOLUZIONI

Scelto il sistema di riferimento locale in figura si ha rispettivamente:

Problema 1

Equazioni della linea elastica

$$\begin{cases} EA\,w^{II}(z)=0 \\ EI\,v^{IV}(z)=+p \end{cases} \Rightarrow \begin{cases} w^{II}(z)=0 \\ v^{IV}(z)=\dfrac{p}{EI} \end{cases}$$

Condizioni al contorno in **a**

$$\begin{cases} w(0)=0 \\ v(0)=0 \\ \varphi(0)=0 \end{cases} \Rightarrow \begin{cases} w(0)=0 \\ v(0)=0 \\ -v^{I}(0)=0 \end{cases} \Rightarrow \begin{cases} w(0)=0 \\ v(0)=0 \\ v^{I}(0)=0 \end{cases}$$

Condizioni al contorno in **b**

$$\begin{cases} N(l)=0 \\ T(l)=0 \\ M(l)=0 \end{cases} \Rightarrow \begin{cases} EAw^{I}(l)=0 \\ -EIv^{III}(l)=0 \\ -EIv^{II}(l)=0 \end{cases} \Rightarrow \begin{cases} w^{I}(l)=0 \\ v^{III}(l)=0 \\ v^{II}(l)=0 \end{cases}$$

Dalle precedenti si ricava:

$$w(z)=A_1\,z + A_2;\ A_1=A_2=0$$

$$v(z)=\frac{pz^4}{24EI}+C_1\frac{z^3}{6}+C_2\frac{z^2}{2}+C_3 z+C_4$$

$$c2 \to \frac{l^2\,p}{2\,EI},\ c3 \to 0,\ c4 \to 0,\ c1 \to -\frac{l\,p}{EI}$$

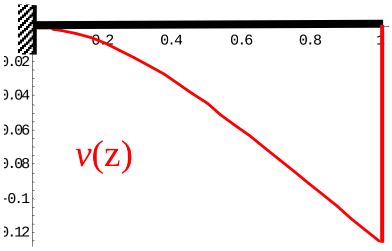
quindi:

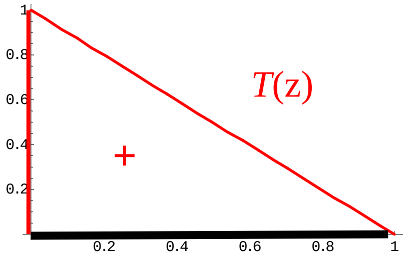
$v(z)=\ \dfrac{l^2\,p\,z^2}{4\,EI} - \dfrac{l\,p\,z^3}{6\,EI} + \dfrac{p\,z^4}{24\,EI}$

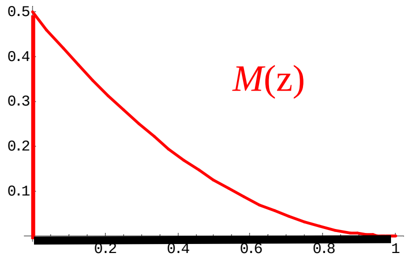
$\varphi(z)= -v^{I}(z)=\ -\dfrac{p\,z\,(3\,l^2 - 3\,l\,z + z^2)}{6\,EI}$

$T(z)= -EIv^{III}(z)=\ p\,(l - z)$

$M(z)= -EIv^{IV}(z)=\ -\dfrac{1}{2}\,p\,(l - z)^2$







Problema 2

Equazioni della linea elastica

$$\begin{cases} EA\,w^{II}(z)=0 \\ EI\,v^{IV}(z)=+p \end{cases} \Rightarrow \begin{cases} w^{II}(z)=0 \\ v^{IV}(z)=\dfrac{p}{EI} \end{cases}$$

Condizioni al contorno in **a**
$$\begin{cases} w(0)=0 \\ T(0)=0 \\ \varphi(0)=0 \end{cases} \Rightarrow \begin{cases} w(0)=0 \\ -EIv^{III}(0)=0 \\ -v^{I}(0)=0 \end{cases} \Rightarrow \begin{cases} w(0)=0 \\ v^{III}(0)=0 \\ v^{I}(0)=0 \end{cases}$$

Condizioni al contorno in **b**
$$\begin{cases} v(l)=0 \\ N(l)=0 \\ M(l)=0 \end{cases} \Rightarrow \begin{cases} v(l)=0 \\ EAw^{I}(l)=0 \\ -EIv^{II}(l)=0 \end{cases} \Rightarrow \begin{cases} v(l)=0 \\ w^{I}(l)=0 \\ v^{II}(l)=0 \end{cases}$$

Dalle precedenti si ricava:

$$w(z)=A_1\,z + A_2;\ A_1=A_2=0$$

$$v(z)=\frac{pz^4}{24EI}+C_1\frac{z^3}{6}+C_2\frac{z^2}{2}+C_3z+C_4$$

$$c4 \to \frac{5\,l^4\,p}{24\,EI},\ c2 \to -\frac{l^2\,p}{2\,EI},\ c3 \to 0,\ c1 \to 0$$

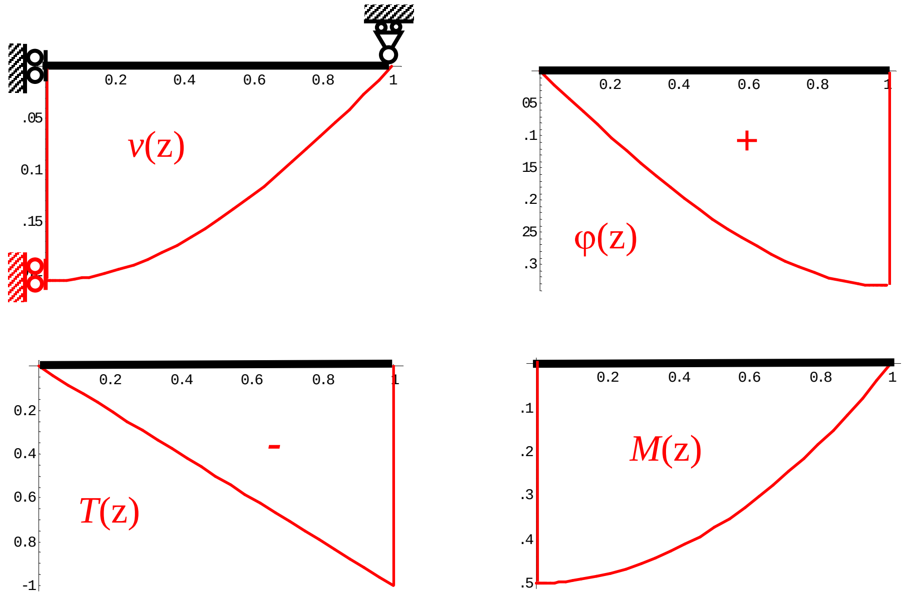
quindi:

$$v(z)= \frac{5\, l^4\, p}{24\, EI} - \frac{l^2\, p\, z^2}{4\, EI} + \frac{p\, z^4}{24\, EI}$$

$$\varphi(z)= -v^{I}(z)= -\frac{p\, z\, (-3\, l^2 + z^2)}{6\, EI}$$

$$T(z)= -\,EIv^{III}(z)= -p\, z$$

$$M(z)= -EIv^{II}(z)= \frac{1}{2}\, p\, (l^2 - z^2)$$



Problema 3

Equazioni della linea elastica

$$\begin{cases} EA\, w^{II}(z)=0 \\ EI\, v^{IV}(z)=+p \end{cases} \Rightarrow \begin{cases} w^{II}(z)=0 \\ v^{IV}(z)=\dfrac{p}{EI} \end{cases}$$

Condizioni al contorno in **a**
$$\begin{cases} w(0)=0 \\ v(0)=0 \\ \varphi(0)=0 \end{cases} \Rightarrow \begin{cases} w(0)=0 \\ v(0)=0 \\ -v^{I}(0)=0 \end{cases} \Rightarrow \begin{cases} w(0)=0 \\ v(0)=0 \\ v^{I}(0)=0 \end{cases}$$

Condizioni al contorno in **b**
$$\begin{cases} N(l)=0 \\ v(l)=0 \\ \varphi(l)=0 \end{cases} \Rightarrow \begin{cases} EAw^{I}(l)=0 \\ v(l)=0 \\ -v^{I}(l)=0 \end{cases} \Rightarrow \begin{cases} w^{I}(l)=0 \\ v(l)=0 \\ v^{I}(l)=0 \end{cases}$$

Dalle precedenti si ricava:

$$w(z)=A_1\, z + A_2;\; A_1=A_2=0$$

$$v(z)=\frac{pz^4}{24EI}+C_1\frac{z^3}{6}+C_2\frac{z^2}{2}+C_3 z+C_4$$

$$c1\to -\frac{l\,p}{2\,EI},\; c2\to \frac{l^2\,p}{12\,EI},\; c4\to 0,\; c3\to 0$$

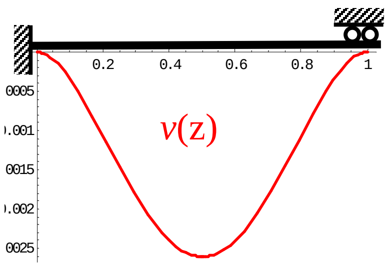
quindi:

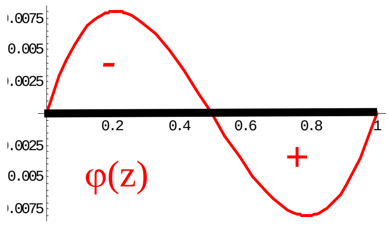
$$v(z)=\frac{l^2\,p\,z^2}{24\,EI}-\frac{l\,p\,z^3}{12\,EI}+\frac{p\,z^4}{24\,EI}$$

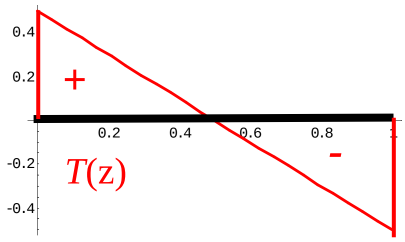
$$\varphi(z)= -v^{I}(z)= -\frac{p\,z\,(l^2-3\,l\,z+2\,z^2)}{12\,EI}$$

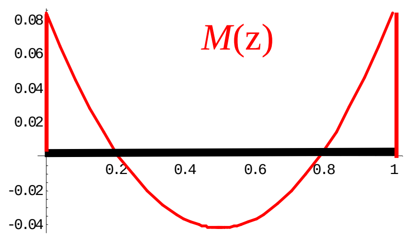
$$T(z)= -\,EIv^{III}(z)= \frac{1}{2}\,p\,(l-2\,z)$$

$$M(z)= -EIv^{II}(z)= -\frac{1}{12}\,p\,(l^2-6\,l\,z+6\,z^2)$$









Problema 4

Equazioni della linea elastica

$$\begin{cases} EA\,w^{II}(z)=0 \\ EI\,v^{IV}(z)=0 \end{cases} \Rightarrow \begin{cases} w^{II}(z)=0 \\ v^{IV}(z)=0 \end{cases}$$

Condizioni al contorno in **a**

$$\begin{cases} w(0)=0 \\ v(0)=0 \\ \varphi(0)=0 \end{cases} \Rightarrow \begin{cases} w(0)=0 \\ v(0)=0 \\ -v^{I}(0)=0 \end{cases} \Rightarrow \begin{cases} w(0)=0 \\ v(0)=0 \\ v^{I}(0)=0 \end{cases}$$

Condizioni al contorno in **b**

$$\begin{cases} v(l)=-\delta_b \\ N(l)=0 \\ M(l)=0 \end{cases} \Rightarrow \begin{cases} v(l)=-\delta_b \\ EAw^{I}(l)=0 \\ -EIv^{II}(l)=0 \end{cases} \Rightarrow \begin{cases} v(l)=-\delta_b \\ w^{I}(l)=0 \\ v^{II}(l)=0 \end{cases}$$

Dalle precedenti si ricava:

$$w(z)=A_1 z + A_2;\ A_1=A_2=0$$

$$v(z)=C_1\frac{z^3}{6}+C_2\frac{z^2}{2}+C_3 z+C_4$$

$$c1\to\frac{3\,\delta_b}{l^3},\ c2\to-\frac{3\,\delta_b}{l^2},\ c3\to 0,\ c4\to 0$$

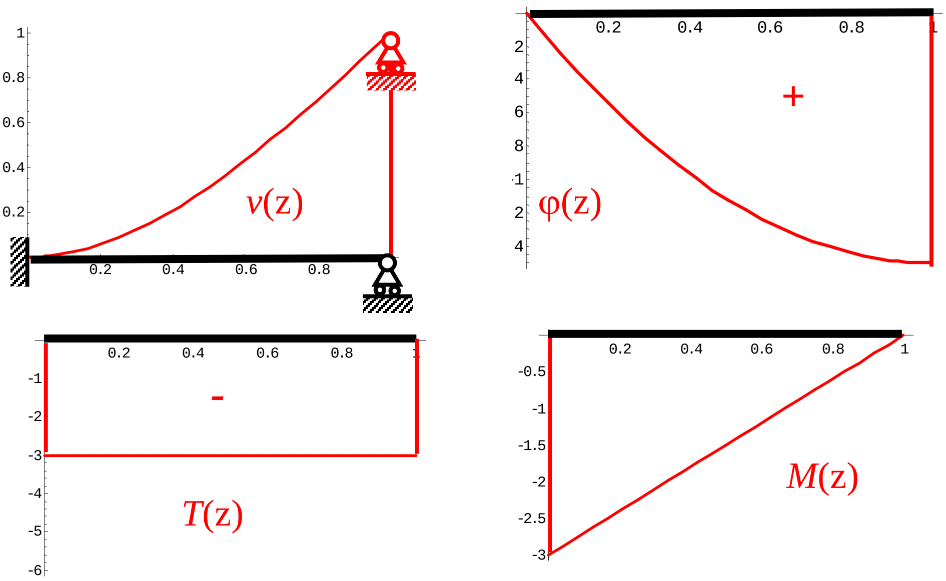
quindi:

$$v(z)= -\frac{3\,z^2\,\delta_b}{2\,l^2}+\frac{z^3\,\delta_b}{2\,l^3}$$

$$\varphi(z)= -v^{I}(z)= \frac{3\,(2\,l-z)\,z\,\delta_b}{2\,l^3}$$

$$T(z)= -\,EIv^{III}(z)= -\frac{3\,EI\,\delta_b}{l^3}$$

$$M(z)= -EIv^{II}(z)= \frac{3\,EI\,(l-z)\,\delta_b}{l^3}$$


v(z)
φ(z)
+
T(z)
-
M(z)